Wirelessly Energy Harvesting DF Dual-hop Relaying Networks: Optimal Time Splitting Ratio and Performance Analysis

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Abstract—This paper is to derive the optimal time splitting ratio for wireless energy harvesting DF dua-hop relaying networks. Using the partial relay selection, the best relay having the largest harvesting energy is chosen to be the forward for the second hop. We also derive the closed form expression for the optimal time splitting ratio that maximizes the system instantaneous capacity. Numerical results have shown that the system with the optimal time splitting ratio can significantly improve instantaneous and ergodic system capacity at high signal-to-noise ratios for the same channel and system settings.

Index Terms—Energy harvesting, harvest-use architecture, partial relay selection, optimal time split, Rayleigh fading channel, outage probability.

1. Introduction

Extending wireless network lifetime by using energy harvesting (EH) technique is a practical solution, especially for wireless sensor networks. With EH, operating power in each wireless sensor node can be refilled from many sources, such as wind, solar, thermoelectric effects or other physical phenomena [1], [2]. Recently, a promising solution that energy absorbed from radio frequency (RF) waves distributed by surrounding transmitters has been proposed by Varshney [3]. The advantage of this solution lies on the fact that we can collect energy as well as information simultaneously. This promising idea can be brought into practical implementation with feasible receiver designs, e.g., [4]–[8]. EH systems can be mainly classified into two kinds of systems including (i) Harvest-Use (HU): the harvested energy is used at once and cannot be accumulated for future use and (ii) Harvest-Store-Use (HSU): the energy is firstly harvested, then it can be stored for later utilization [9], [10].

Recently, many recent studies have focused on two-hop EH relay communication systems. For single and multiple relay scenarios, several EH relay systems have been investigated, see, e.g., in [11]–[15]. In particular, Gunduz et al. proposed an optimal offline transmission scheme for both full-duplex and half-duplex transmission modes that maximize the total amount of data transmitted to the destination in a constraint time deadline [11]. In [12], the authors shown that cooperative communication using energy buffer can provide a higher maximum stable throughout than direct communication in the constraint of poor energy arrival rates. Considering two-hop relaying systems with an EH source helped by a non-EH relay, Luo et al. proposed an optimal transmission policy confirming the importance of the adaptive power allocation in EH relay networks [13]. For multiple EH cooperative amplify-and-forward (AF) relays, paper [14] has studied cooperative wireless networks with voluntary energy harvesting relays for cases, where the signal-to-noise-ratio or the number of relays is large showing that EH relays in cooperative communication can provide an effective solution for improving performance. Very recently, Do et al. proposed a novel derivation approach to derive closed-form approximations of system outage probability for EH based dual-hop decode-and-forward (DF) relaying networks under partial relay selection and optimal relay selection [15].

In all EH based wireless networks with the time splitting method, time splitting ratio is the key system parameter determining system performance. To the best of the authors knowledge, the optimal problem of time splitting ratio in harvesting process has not been fully investigated, except for the paper [16]. In [16], Krikidis et al. provided an approximated value of the optimal time split that maximizes the instantaneous capacity in HU AF relaying networks. However, the result in this paper is restricted on a single relay.

Different from the above related works, this paper focuses on a two-hop DF relay network with multiple EH relays operating on HU architecture. Partial relay selection protocol is adopted for the second hop, i.e., only the selected relay, which has the best harvested energy, forwards the re-encoded version of the received signal from the source towards the destination [17],
We also derive the closed form expression for the optimal time splitting ratio to maximize the instantaneous system capacity. We use Monte Carlo simulation and numerical approach to verify the obtained analysis results. Numerical results show that optimal time splitting ratio can provide the best instantaneous and ergodic system capacity as compared with fixed and randomized cases.

The remaining parts of this paper are organized as follows. In Section 2, the system model and the system operation description are provided. The optimal problem of time splitting ratio is formulated and solved in Section 3. The analytical and empirical OP results are provided in Section 4. Finally, Section 5 provides some concluding remarks of this paper.

2. System model

![Dual-hop DF relay network with multiple EH relays and partial relay selection.](image)

We consider a half-duplex (HD) network as shown in Fig. 1. The communication between the source S and the destination D is helped by a set of N relays, denoted as R_i with i = 1, . . . , N. Both the source and the destination are non-EH nodes, whereas all relays are EH nodes, i.e., they have to harvest energy from the source RF signals to support data transmission. We assume that there does not exist the direct link between the source and the destination due to shadowing and pathloss effect. We further assume that all nodes are equipped with one antenna each.

The system works in two modes on each fixed block time T including energy harvesting mode (EHm) and data transmission mode (DTm). Under the time switching architecture [4], two sub-timeslots, i.e., energy harvesting and information transfer with corresponding time durations αT and (1 − α)T, take turns consecutively, where α denotes the time splitting ratio. α is an adjustable design parameter taking value from 0 to 1.

In the first sub-timeslot, all relays harvest energy from the source. In the second sub-timeslot, partial relay selection is used, i.e., the relay having highest instantaneous gain or equivalently highest harvested energy, denoted as R^∗, is the forwarder of the second hop [17], [19], [20]. For DF relaying, the selected relay will fully decode the received signal, re-encode, and then forward the resulting signal toward the destination.

Let h_i with i = 1, . . . , N, h^∗, and g denote the channel coefficients for the links S → R_i, S → R^∗, and R^∗ → D, respectively. Assume that all narrowband links are subject to flat, independent but non-identical Rayleigh fading and additive white Gaussian noise (AWGN) with variance N_0. Over Rayleigh fading channels, the channel power gains, i.e., ||h_i||^2 and ||g||^2, are exponentially distributed with means λ_h and λ_g, respectively, ∀i. We can write the signal-to-noise ratios (SNRs) of the links S → R^∗ and R^∗ → D, respectively, as follows:

\[
\gamma_1 = \frac{P_S ||h^*||^2}{N_0}, \quad (1)
\]

\[
\gamma_2 = \frac{P_{R^*} ||g||^2}{N_0}. \quad (2)
\]

The energy harvested by R_i in the EHm mode can be expressed as

\[
E_H = \eta P_S ||h_i||^2 \alpha T, \quad (3)
\]

where η is the energy conversion efficiency with 0 < η < 1.

During the second sub-timeslot of (1−α)T/2, R^∗ will transmit its data with the transmit power P_{R^*} given by

\[
P_{R^*} = 2\eta P_S ||h^*||^2 \frac{\alpha}{1−\alpha}. \quad (4)
\]

Let γ_{e2e} be the overall SNR of the proposed system. As being proved in [21] and [22], this γ_{e2e} is limited by the weakest hop regardless of modulation scheme employed. Mathematically, γ_{e2e} can be tightly approximated as follows:

\[
\gamma_{e2e} = \min(\gamma_1, \gamma_2), \quad (5)
\]

where \gamma_2 are rewritten as follows:

\[
\gamma_2 = \frac{2\eta P_S \alpha}{N_0(1−\alpha)} ||h^*||^2 ||g||^2. \quad (6)
\]

The system ergodic capacity can be written as

\[
C = \frac{1−\alpha}{2} \log_2 \left( 1 + \min \left( \frac{P_S ||h^*||^2}{N_0}, \frac{2\eta P_S \alpha}{N_0(1−\alpha)} ||h^*||^2 ||g||^2 \right) \right), \quad (7)
\]

where the pre-factor of (1 − α)/2 is account for the data transmission.

3. Optimal time splitting ratio problem

In energy harvesting DF dual-hop relaying networks, α is one of the key system parameters, which determines the system performance. If the value of α is set too small, the relay can not harvest enough energy to support data transmission. Otherwise, the system can not support the desired data rate due to time limitation. To provide the fundamental trade-off between EH time and communication time, we define the optimal problem of time splitting ratio as follows:

\[
\alpha_{opt} = \arg \max_{\alpha \in [0,1]} \frac{1−\alpha}{2} \log_2 \left( 1 + \min (\gamma_1, \gamma_2) \right). \quad (8)
\]
For the current form of (8), it is difficult to derive the optimal solution by direct making derivative of the function with respect to $\alpha$. It is convenient to consider two separate cases of $\gamma_1$ and $\gamma_2$, i.e., Case 1: $\alpha \leq \alpha_0$ and Case 2: $\alpha > \alpha_0$, where

$$\alpha_0 = \frac{1}{1 + 2\eta|g|^2}$$  \hspace{1cm} (9)

by solving $\gamma_1 = \gamma_2$.

- **Case 1**: For given $|h^*|^2$ and $|g|^2$ and considering $\alpha \leq \alpha_0$, we can rewrite (18) as

$$C(\alpha) = \frac{1 - \alpha}{2\log 2} \log \left( 1 + \frac{2\eta P_3 \alpha}{N_0(1 - \alpha)} |h^*|^2 |g|^2 \right).$$  \hspace{1cm} (10)

Likewise, taking the derivative of $C(\alpha)$ in (10) with respect to $\alpha$, i.e., $\frac{dC(\alpha)}{d\alpha} = 0$, we have

$$\log \left( 1 + \frac{\rho \alpha}{1 - \alpha} \right) = \frac{\rho}{1 - \alpha + \rho \alpha},$$  \hspace{1cm} (11)

where $\rho = \frac{2\eta P_3 |h^*|^2 |g|^2}{N_0}$.

We rewrite (11) as follows:

$$1 + \frac{\rho \alpha}{1 - \alpha} = e^{1 - \eta = \frac{\eta}{\alpha + \rho \alpha}},$$  \hspace{1cm} (12)

Making a change of variable, i.e., $u = \frac{\rho}{1 - \alpha + \rho \alpha}$, we have

$$(u - 1)e^{u - 1} = \frac{\rho - 1}{e}.$$  \hspace{1cm} (13)

We can see that (13) is of the standard form of LambertW function, which has a solution as $u = 1 + \mathcal{W}(\frac{\eta}{\rho})$ [23] leading to

$$\alpha^*_1 = \frac{1 - \frac{\rho}{1 - \eta \mathcal{W}(\frac{\eta}{\rho})}}{1 - \rho},$$  \hspace{1cm} (14)

where $\mathcal{W}(\cdot)$ is the Lambert W-function [24].

- **Case 2**: We start this case by considering the condition of $\gamma_1 < \gamma_2$, i.e., $\alpha_0 \leq \alpha \leq 1$, (8) is rewritten as

$$C(\alpha) = \frac{1 - \alpha}{2\log 2} \log \left( 1 + \frac{P_3 |h^*|^2}{N_0} \right).$$  \hspace{1cm} (15)

It can be easily observed that $C(\alpha)$ is decreasing on the interval of $[\alpha_0, 1]$. Therefore, the instantaneous capacity will be maximized, when $\alpha$ takes the value of $\alpha_0$, i.e.,

$$\alpha^*_2 = \alpha_0.$$  \hspace{1cm} (16)

It is noted that if $\alpha^*_1$ is greater than $\alpha_0$, $\alpha^*_1$ is not a valid root for (13). As a result, $\alpha_{op} = \alpha^*_2$. Combining (14) and (16), we finally obtain the closed form expression for the time splitting ratio as follows:

$$\alpha_{op} = \begin{cases} 
\alpha^*_1, & |g|^2 < \frac{1}{2\eta} \left( \frac{\rho \left( 1 + \mathcal{W}(\frac{\eta}{\rho}) \right)}{1 - \mathcal{W}(\frac{\eta}{\rho})} \right) \\
\alpha^*_2, & \text{otherwise}
\end{cases}.$$  \hspace{1cm} (17)

Having $\alpha_{op}$ at hands, we can write the system ergodic capacity as follows:

$$\overline{C} = \frac{1 - \alpha}{2} \int_0^\infty \log_2 \left( 1 + \gamma \right) \frac{1}{\alpha_{op}} d\gamma.$$  \hspace{1cm} (18)

### 4. Numerical results

The purpose of this section is to verify the above analysis results. In all simulation cases, we set $\lambda_s = \lambda_g = 1$ except stated otherwise and the energy conversion efficiency $\eta = 0.5$.

In Fig. 2, the system instantaneous capacity is studied for different time splitting ratios. We assume that channel CSI is known at respective transmitters, i.e., $|h^*|^2 = 1.9948$ and $|g|^2 = 1.0246$. Three cases of transmit powers of the source are investigated. The solid line is drawn in case of $P_3 = 0$ dB, the dash line is for $P_3 = 10$ dB, and the last one represents for $P_3 = 20$ dB. As we can see the optimal value for $\alpha$ of three considered cases, i.e., 0, 10, and 20 dB will be 0.4939, 0.3630, and 0.2364, respectively. As expected, increasing the source average transmit power will decrease $\alpha_{op}$. It is due to the fact that all relays will need less time to harvest energy for data transmission. As a check, we use numerical approach in Matlab with function $\text{fminbnd}$ to obtain the same optimal value of $\alpha$ confirming the correctness of the analysis results.

Fig. 3 shows the ergodic capacity versus average SNRs. Three cases of $\alpha$ are considered, i.e., Case 1: $\alpha$ fixed at 1/3; Case 2: $\alpha$ chosen randomly in range of $[0, 1]$; Case 3: $\alpha$ optimally calculated using (17). As clearly shown in the Fig. 2 that the optimal time splitting ratios give the best performance. Whereas, the randomized $\alpha$ gives the worst ergodic capacity. Furthermore, the optimal $\alpha$ can provide significant capacity gain at high SNR regime.

In Fig. 4, we investigate the effect of number of relays on the system ergodic capacity. As we can see, increasing the number of EH relays can improve the system capacity but the diminishing returns are obtained as the number of relays increases. For example, when $N$
increases from 1 to 4, 3 dB gain of ergodic capacity is obtained, but only 1 dB gain of capacity is achieved when we continuously increase $N$ from 2 to 3 or 3 to 4. It is due to the effect of partial relay selection, as expected.

In Fig. 5, we investigate the effect of relay position on the system ergodic capacity. Using the simplified pathloss model, i.e., $\lambda_h = d^{-\beta}$ and $\lambda_h = (1-d)^{-\beta}$, where $d$ denotes the physical distance from the source to relays and $\beta$ is the pathloss exponent. Here, we set $\beta = 4$ for illustrative purpose. As we can see in Fig. 5, system ergodic capacity will decrease if $d$ increases if the distance between the source and the destination is normalized by one. It is because $\alpha$ is optimally adjusted to balance between two sub-timeslots of energy harvesting and information transfer.

5. Conclusion

In this paper, we proposed and analytically solved the optimal time splitting ratio for EH dual-hop relaying network with partial relay selection. Monte Carlo simulation is used to verify the analysis results, which also confirm the correctness of our derived results. Numerical results show that the optimal time splitting ratio can provide the best system ergodic capacity, especially at high SNR regime. In addition, partial relay selection can improve the system capacity in all range of average SNRs. As a future work, we will study the effect of multi antenna equipped on the destination on the system performance.

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References


